

# Energy efficiency in extrusion-related polymer processing: a review of state of the art and potential efficiency improvements

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## Abstract

Energy saving and industrial pollution have become increasingly important issues, therefore the identification and adoption of more energy efficient machines and industrial processes are now industrial priorities, and worthy topics for further development through academic research. Polymeric materials are a major raw material, finding widespread application to a range of current industrial machine components as well as multiple products and packaging found in our daily life. Polymer extrusion serves as a particular example of polymer processing techniques, representative of others in as much as there are analogous intermediate stages in the processing. Processing techniques which require such intermediate stages include the manufacture of blown film, blow moulding, thermo-forming, and injection moulding. Hence, the study of polymer extrusion is a representative paradigm for a wider range of processing techniques. Since polymer processing is an energy intensive process and accounts for a huge share (maybe more than 1/3) of the materials processing sector, any improvement to the process would contribute significantly to global energy savings. This work presents a review of studies, which focus on, or appertain to, the energy consumption of extrusion related polymer processing applications. Typical energy demand and losses during processing are considered, and possible approaches for improving the process energy efficiency while maintaining the required end product quality are considered. Overall, this work provides a detailed discussion about how and where energy is utilized; how, where and why energy losses occur; and sets out approaches for optimizing the process energy efficiency.

### Keywords:

Energy consumption, Energy losses, Energy savings, Polymer extrusion, Process monitoring, Process control, Materials processing, Energy efficiency, Industry 4.0, Circular economy, Dynamical systems

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## 1. Introduction

### 1.1. Market demand for polymers

As the number of applications for polymer materials in high volume manufacturing sectors, such as packaging, continues to grow, it is timely to consider manufacturing process optimisation from the energy efficiency point of view. At the present time, the increasing adoption of thermoplastics for use in high performance component applications, such as automotive and aerospace, has meant that product quality has been the prime focus of process optimisation. As the manufacturing processes associated with polymer processing have become more mature, there has been a correspondingly greater utilisation of in-line sensors, and adoption of Industry 4.0 protocols. This has enabled greater understanding of the material performance under processing temperatures and pressures, thereby providing the necessary input data needed for high fidelity computational modelling. In the field of computational optimisation, there have been significant advances in algorithms development, with the result that a much bigger class of multi-variable and

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multi-objective problems can be addressed. As a result, the possibility to broaden the scope of process optimisation can now be grasped.

Clearly, for both high volume and high performance applications, energy efficient manufacture is a desirable goal, which not only leads to reduced manufacturing costs but also addresses National and International energy and  $CO_2$  reduction targets. As a result, scrutiny of the energy required in an energy intensive manufacturing process such as the extrusion process is driven by both business and environmental imperatives. The payment of energy bills for unnecessary usage reduces profit margins and hence increases the end product/service prices for customers. Meanwhile, because  $CO_2$  emissions are now very clearly understood to be detrimental to the environment, energy usage will be increasingly subject to disincentives such as high fuel commodity pricing and taxation.

The scale of the plastics industry is internationally huge, and expanding. For example, in 2015 in the UK [1], there were of the order of 6,200 plastics companies, employing nearly 170,000 people, and with a combined annual sales turnover of over £23.5 bn, of which one third represented exports. According to the reports of PlasticsEurope [2], by the year 2016 the European plastics industry comprised of more than 60,000 companies, employing more than 1.5 million people, and with total sales exceeding 350 EUR bn. Globally, plastics production has grown from 204 to 335 million tonnes between 2002 and 2016. The statistics presented in Figures 1 and 2 illustrate this growing demand.

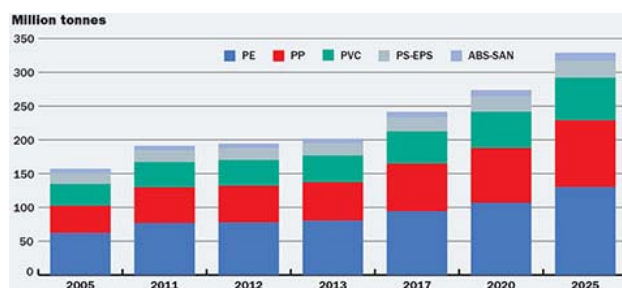


Figure 1: Major thermoplastics: World demand distribution, by polymer between years 2005-2025 [3]

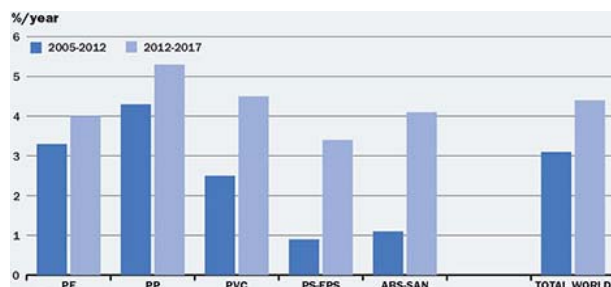


Figure 2: Major thermoplastics: World consumption growth rate, by polymer (2005-2012 and 2012-2017)[3]

Given this level is sustained, the level of growth in demand, and the development and accessibility of new polymer processing capability, it is clear that improvements in process energy efficiency could have a significant impact on global energy savings [4, 5]. Furthermore, the European Best Practice Guide [6] claims, “Plastics are the material for the 21<sup>st</sup> century”, explaining that a 3 Megatonne  $CO_2$  emission reduction could be achieved in Europe by a 10% reduction in the plastics industry energy consumption. With the current capacity of polymers and plastics manufacturing sector, it is one of the largest energy consumers in industrial manufacturing and also a major source of global waste generation. Meantime, the energy savings/optimization in the manufacturing sector is considered as one of the main pillars of modern circular economy concept and both manufactures and consumers have been forced to re-think the current take-make-waste extractive industrial model for reusing materials from end-of-life components/devices, where polymers/plastics industry is one of the major focuses of this concept [7].

## 1.2. The polymer extrusion process

A polymer “extruder” machine processes materials by forcing them through a set of processing stages. The operation and basic processing stages are described in Figure 3 below. The screw passes material through a cylindrical

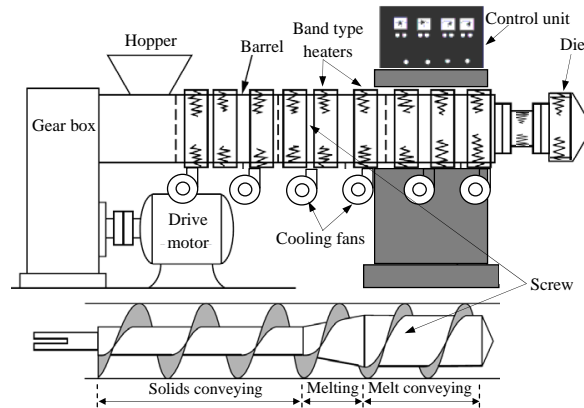


Figure 3: Operational schematic of a single screw extruder

barrel, around which heaters are wrapped, to provide the necessary heat for material melting. In addition to this externally provided heat, a significant amount of heat is generated internally, inside the barrel, as a result of the mechanical work of the screw (*i.e.* the work done against viscous and frictional forces). The feed material absorbs heat as it is conveyed along the screw and is expected to be in the fully molten state at the point that the molten material is forced into a die to form into the desired shape.

Currently, different types of extruders (*e.g.* single screw, multi screw, and disc/drum types) are available in industry, while screws with different geometrical designs are commercially available. Moreover, a number of process monitoring devices are used, to observe process functionality, and for diagnosing possible processing problems.

Frankland [8], President of *Frankland Plastics Consulting, LLC*, explains this in detail in his on-line article about estimating extrusion melt temperature. The most significant points are that the mechanical energy feed into the drive is converted by the screw action on the material to create heat, and thus melting of the polymer. The energy share required for material conveying is relatively smaller, as is the energy supplied to the barrel heaters. He also lists energy losses and their sources. More details on the polymer extrusion process and its operation can be found in the literature [9, 10, 11].

Manufacturing process stability is a key concern, and variation in the material temperature presents a challenge to the end product quality control. For this reason most commercial polymer producers avoid operating their extruders at the higher screw speeds. This is unfortunate since at higher speeds, and therefore at higher workpiece temperatures, there is more potential for process energy efficiency improvement because the material viscosity is reduced and thus the forming forces required are lower. Moreover, this undesirable cost is repeated, since many thermoplastic polymers are extruded more than once before their final products are manufactured [12]. Better concatenation of extrusion process steps would lead to greater energy reduction, by maintaining or controlling the heat in the workpiece during processing, thereby avoiding the need to re-heat.

### 1.2.1. Basic processing mechanisms

Zones within the polymer processing screw can be broadly designated, based on the functional activity taking place within that zone, see Figure 3. The points of transition between zones are not generally well defined, as they depend on the processing conditions and the materials.

#### a. Solids conveying

In this zone, the polymer is preheated before passing into the subsequent zones. While flowing along this zone, material starts to absorb heat from the barrel heaters, but the mechanical heat generated by frictional and viscous

mechanisms is dominant in this zone [13, 14, 10, 15]. Generally, the screw channel depth is maintained constant in order to provide a constant material feed to the subsequent zones.

The first comprehensive theory for the action of solids conveying was developed by Darnell and Mol [16] in the 1950s and this quantitative description still remains as the widely accepted model for solids conveying in extrusion.

### b. Melting or Plastication

Experiments for studying the polymer extrusion melting mechanism were first carried out by Maddock and Street in 1959 [17]. The melting mechanism proposed by Maddock for single screw extruders still remains as the most widely accepted melting mechanism in polymer extrusion. The Maddock melting mechanism is only a qualitative description of melting which occurs in single screw extruders. Maddock used a visual inspection method to investigate the melting process by stopping the screw rotation suddenly during the process and 'freezing' the polymer by cooling the barrel and screw rapidly. Later, Tadmor also extended the understanding of melting mechanism of extrusion processes [18, 19, 20, 21].

As the material reaches the "end" of the solids conveying zone, it begins to melt, and as such is considered to have entered into the melting zone. As the material becomes soft, further heat will be added to the process by means of viscous dissipation of the material (*i.e.* work done against the viscoelastic nature of the material). Both solid and molten polymers co-exist in this zone. The solid bed would comprise both compacted solid polymer abutting the "trailing flight", and the melt pool pushing against the "pushing flight", as shown in Figure 4. As the material

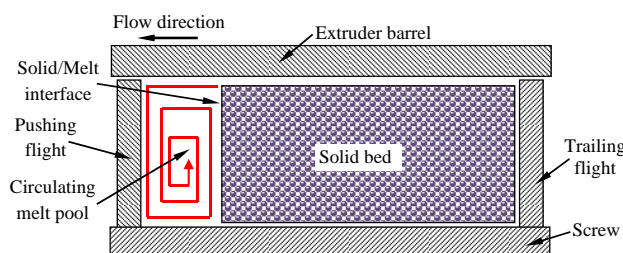


Figure 4: An illustration of the typical arrangement of the solid bed and melt pool inside a screw channel for a single-flighted conventional screw

proceeds along the screw, the proportion of melt pool to solid bed increases. The screw channel depth is therefore designed to become smaller, which influences flow rate and mixing; however, the actual screw length at which melting occurs depends on a range of parameters such as screw geometry, operating conditions and physical properties of the polymer [20].

As was claimed by Severs [22], the plastication or melting process has a direct impact on the quality of the material properties of final product, and thus must be carefully controlled. Tadmor, Klein and Gogos [18, 19, 21] proposed an equation for calculating the rate of melting, ( $\Omega$ ), in a screw channel, and is given by Eq. (1).

$$\Omega = \left[ \frac{\rho_m \times V_{bx} \left\{ k_m (T_b - T_{melt}) + \eta \left( \frac{V_j^2}{2} \right) \right\}}{2 \{ C_p (T_m - T_s) + \lambda \}} \right]^{1/2} \quad (1)$$

Where  $\rho_m$  is the melt density,  $V_{bx}$  is the transverse component of the barrel velocity,  $k_m$  is the thermal conductivity of the molten material,  $T_{melt}$  is the melt temperature,  $T_b$  is the barrel temperature,  $\eta$  is the melt viscosity,  $V_j$  is the resultant relative velocity,  $C_p$  is the polymer specific heat capacity, and  $\lambda$  is the temperature of the solid bed.

This equation clearly demonstrates that the melting rate can be increased by increasing the screw rotational speed [14]; however, for higher speeds, the polymer passes through more quickly, giving less time for temperature stabilisation, and thus more variation in melt viscosity [10, 14]. As a result, to ensure controlled plastication, it is necessary to control the melting rate, and this in turn depends on the material being processed, process set conditions, and nature of the processing unit/machine [23].

### c. Melt conveying

Melt conveying starts as complete melting is achieved. The screw channel depth is constant along the zone and is shallower than in the other two zones. During this stage further heating and mixing of the melt takes place as the polymer is smeared by the tip of the screw flight against the barrel wall. Material has to be moved towards the die with enough force to overcome the head pressure generated at the die – this is known as the “die head pressure”.

Melt output rate from this zone depends on a combination of two main factors: the rate of the rotation of the screw and the screw channel pressure gradient [24]. Proper mixing of material is another requirement for the flow through this zone. The melt conveying zone of some of the new screw designs is fitted with efficient mixer units to ensure good mixing performance (*e.g.* the barrier flighted screw with a Maddock mixer).

Studies on melt conveying operation of extrusion were reported very much earlier than in the other two zones. One of the initial studies was carried out in 1920s [25, 26], which proposed the calculation of the melt conveying rate by considering the melt flow as a laminar fully developed flow. This is still a widely accepted model.

In addition to the above mentioned mechanism/theories, several other works have been reported later on improving the understanding of these three main mechanisms and more details can be found in the literature [14, 9, 10, 11, 27].

## 2. Energy required for materials processing

The assessment of energy requirements is not straight forward. The overall extrusion process can be broken down into smaller activities, but even then, the power demands at each stage depend in a complex way on a large number of processing parameters. A useful energy flow model was developed by Severs [22], as presented in Figure 5.

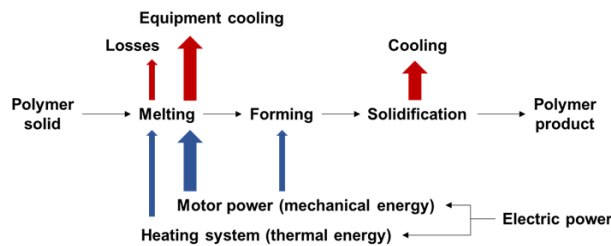


Figure 5: Typical energy flow diagram for an extrusion process

The energy,  $E_u$ , used by an extruder for useful work in material melting and forming, [28], is given by Eq. (2):

$$E_u = E_{in} - E_{losses} \quad (2)$$

where  $E_{in}$  is the energy input to the extruder and  $E_{losses}$  is the energy expended that does not contribute to the extrusion process. Thus, the energy efficiency can be given by Eq. (3):

$$\eta_{extruder} = \frac{E_{in} - E_{losses}}{E_{in}} \times 100\% \quad (3)$$

In these equations, the energy inputs ( $E_{in}$ ) should be related to the energy consumed by the electrical components such as drive motor, barrel/die heaters, barrel/motor cooling fans, water pump/s, instrumentation in the control unit, *etc.*. The energy losses are always associated with all the components and also occur due to forced cooling and via natural convection and radiation, which can be accounted under  $E_{losses}$ . In general, the drive motor and the barrel and die heaters are the source of the highest energy losses. In typical polymer extrusion processes, recovery of such lost energy is impractical, as this is largely released as heat energy to water or air. More details concerning the energy required for polymer processing and the thermodynamic efficiency of an extruder have been discussed by the authors previously [28].

## 3. Prior art in extruder energy evaluation, monitoring and modelling

### 3.1. Energy Consumption studies

In considering the energy consumption in any industrial process, the first step is to review the process capability of the existing or available plant machinery, and the power consumption. On that basis, potential modifications or

enhancements to machinery could be identified, where an economic case could be made on the basis of energy cost saving. Some examples are given below.

In the late 1970s, Chung *et al.* [12] found that for a 63.5 mm diameter extruder mechanical energy efficiency of 62% was typical, and for larger extruders the energy efficiency was lower. In 1981, Kruder and Nunn [29] claimed that energy efficiency of extruders can range from 45%-75%. It was noted that the energy efficiency depended on the transmission mechanism, screw design, product geometry, nature of polymer feedstock and the production rate, while the major energy losses of an extruder occur as a result of the forced cooling process step, and the losses associated with the drive and transmission unit. At low screw speeds, barrel heaters consume a considerably higher portion of energy than at higher speeds, and significant energy savings could be made by running the processes at the highest possible power factor. Additionally, this work presented information on energy demand and losses of each individual component of an extruder.

Subsequently through the 1980s, most research into energy efficiency was focussed on the screw efficiency and mass flow rate. A reduction in the overall power requirement for an extruder can be achieved through the use of a geared pump at the end of the extruder to increase the mass flow rate (McKelvey [30]). In 1985, Strauch *et al.* [31] carried out an energy consumption study on a 63.5 mm diameter single screw extruder, and observed that most of the energy was consumed by the mechanical parts, with less significant levels of consumption in process heating. The energy conversion was then assessed and it was found that heating the water in the cooling system accounted for more than half of the energy supplied.

During the late 1980s and 1990s, the manufacturing sector was making many changes, with a view to improving productivity and quality. Driven by the advances made in Japanese manufacturing, the main focus during that time was on management methods, such as Total Quality Management, LEAN, and Six Sigma. These efforts initially addressed cost and time issues, where the biggest economic benefits were to be found. Latterly, interest in energy efficiency began to be seen as not only cost reduction opportunity but also as an environmental imperative.

In the context of power consumption in the extruder, in 1997 Anderson *et al.* [32] recognised that for the processing of most plastics, from room temperature, the specific energy consumption (SEC) of the extruder motor should be in the range of 0.0822 to 0.1644 kW.hr/kg.

At around the same time, a study by Falkner in 1997 [33] showed that motor operations accounted for over 65% of the 1994 UK industrial electricity usage. Asserting that more than 10% of this energy could be attributed to inefficiency, Falkner argued that this represented a loss of about £0.5 billion to the annual UK economy. These values accounted for motor energy utilisation across multiple industrial sectors, but it should be recognised that the electric motors in plastics industry processing machines are a major power consumers.

A more detailed study by Rosato *et al.* in 2001 [34] observed that energy losses of between 3 and 20% can arise in the transmissions and control systems. Despite this, a conclusion was made that because plastics have lower specific energy requirements compared with most conventional raw materials, they are still highly competitive.

Five years later, Womer *et al.* [35] considered the energy efficiency of extruder cooling. The results demonstrated that water cooling systems consume more energy compared with air cooling, irrespective of the particular plastic being processed. As a result, a recommendation was made to use air only cooling unless extensive cooling was expressly required.

In 2010 [36], the plastics industry was recognised to be one of the major UK industries with a similar trend applying globally. On that basis any improvement in process energy efficiency would lead to a considerable reduction in global energy requirement. Also in 2010, Cantor [37] presented measurements of SEC, where the impact of the motor and of each individual heater zone, with respect to the overall specific energy consumption, was separately recorded. It was observed that the heaters account for over 95% of the supplied energy. In a slightly later study by Heur and Verheijen [38], the authors studied differences from one plant to another, and recommended the use of frequency controllers to enable more precise process control.

The earliest mention of an Industry 4.0 implementation to energy efficiency control was by Jing *et al.* (2014) [39] which proposed the use of real-time monitoring. The rationale was to render unnecessary the installation of power meters or the development of data-driven models. A fuzzy logic controller controlled the high melt quality in a single screw extruder, and was shown to be a cheaper alternative to using a gear pump. This also paved the way for achieving greater extruder energy efficiency by optimising the temperature settings.

A number of other works [35, 40, 41, 42, 43, 44, 45] consider the drive motor efficiency compared with other devices. The conclusion to be drawn is that the drive motor should be the primary design consideration for process

engineering the energy efficiency of the whole extrusion plant.

### 3.2. Influence of process set parameters

In 2001, Rauwendaal [10] recognised the significance of process settings, and presented an account of a procedure to minimise power consumption. A little later, in 2003, Rasid and Wood [46] investigated the influence of individual barrel zone temperatures and found that the solids conveying zone temperature had the greatest influence on overall power consumption.

Studies carried out between 2004 and 2012, [47, 48, 49, 50], examined various process parameters and their influence on SEC. In addition to noting the effect of material viscosity, variation in energy consumption was also seen for different designs of screw, and there were greater melt temperature fluctuations at higher screw speeds: another example of the ever-present tension between cost and quality. The simultaneous need to achieve both energy efficient operation and finished part quality remains a challenge.

Studies carried out by Abeykoon *et al.* [51, 28, 52, 53] between 2009 and 2016, focussed on the relationship between the process energy demands of the motor and barrel heating and melt thermal stability. The effects of the settings for these processes, the screw geometry and choice of material were explored.

### 3.3. Modelling

Following a thorough trawl of the published scientific literature, it has become clear that relatively little work has been undertaken to model extruder energy consumption.

The earliest work in this area was by Mallouk and Mckelvey [54] in 1953, where a mathematical equation was developed, based on assumptions of isothermal, Newtonian flow, in a screw channel with constant section. In 1996, Wilczynski [55] developed a computer model where the five zones of the extruder plus the die were considered separately. Subsequently, in 2000, Lai and Yu [56] also proposed a mathematical model for the calculation of energy consumption based on screw speed, and including viscosity. In Abeykoon *et al.*'s [57, 52] studies of a single screw extruder, the data collected was analysed using static nonlinear polynomial models. The conclusion of the analysis was that choosing energy efficient process settings would also lead to thermal stability.

Obviously, the availability of advanced modelling methods for predicting energy consumption, based on process parameters, would enable process operators to select optimum operating conditions. In particular, models which incorporate both energy consumption and melt thermal quality would be preferred but the development of such models is quite challenging. Melt thermal quality and energy efficiency present opposite behaviours with respect to the processing speed: the thermal quality deteriorates while the energy efficiency improves. Since the industrial sector has to meet strict environmental regulations to minimize the carbon footprint, any reduction in the energy demands for polymer processing would support future sustainability.

### 3.4. General considerations in energy usage

According to basic electricity principles, the typical power consumption of a DC and an AC device ( $P_{DC}$  and  $P_{AC}$ ) is given by equations (4) and (5), respectively [58, 59],

$$P_{DC} = V \times I \quad (4)$$

$$P_{AC} = V \times I \times \cos \phi \quad (5)$$

with  $I$  being the supply current,  $V$  voltage, and  $\cos \phi$  the “displacement power factor”. From these, the power demand of any device in an extrusion plant can be evaluated; however, the energy losses related to each device might vary from component to component.

The power factor is an important consideration in the assessment of the energy usage of an electrical machine or process, and is defined as either the “displacement power factor” which is the  $\cos \phi$  in Eq. (5) or the “true power factor” which is given by Eq. (6).

$$\text{True power factor} = \frac{\text{True (or active) power}}{\text{Apparent power}} \quad (6)$$

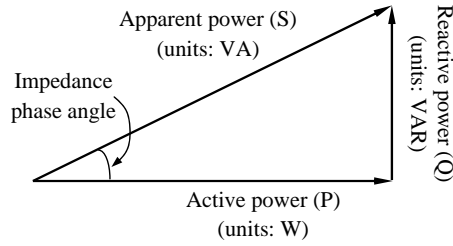


Figure 6: Power triangle showing the relationship between active, apparent and reactive powers

The true power factor lies in the range 0 – 1, for which the running of the machine or process with true power factor equal to one would be the best possible energy efficient operating condition (when the impedance phase angle shown in Figure 6 is equal to zero). For a true power factor of less than one, the energy supplied to the load is not used optimally. In such a case, a higher current must be drawn to compensate for the phase shift,  $\phi$ . Where industrial customers operate with power factors below around 0.95, [60], this represents unbalanced additional power demands from the power supplier, and hence additional infrastructure demand leading to additional costs. Furthermore, electrical devices are attributed with a  $I^2 \times R$  heat loss ( $R$  is the electrical resistance), so that increasing the required current while reducing the power factor results in an increase of power loss as heat.

Measurements of power factor and total power consumption, for a DC motor driven 63.5 mm in diameter single screw extruder operated at different screw speeds, are shown in Figure 7.

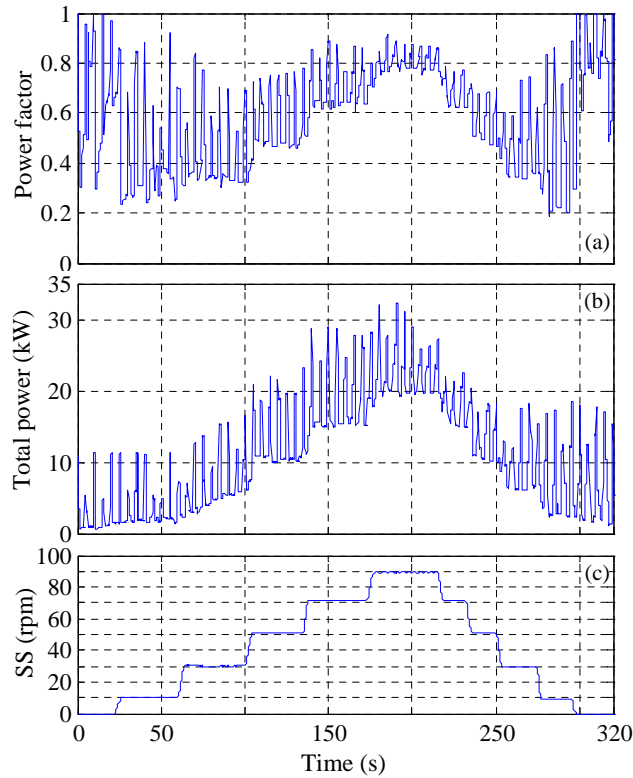


Figure 7: (a). Power factor, (b). Total extruder power, (c). Screw speed [52]

From this, it can be seen that both the power factor and the total power required are greater for greater processing speeds; however, for higher processing speeds, the temperature uniformity of the process melt output deteriorates significantly, leading to poor product quality, [28, 52, 53]. Hence, the running of these processes at higher speeds and



with the highest possible power factor is problematic, despite being desirable for energy efficiency.

#### 4. Potential for energy efficiency improvements

Energy demands and losses are illustrated in the form of an energy flow diagram, Figure 8. This diagram may be

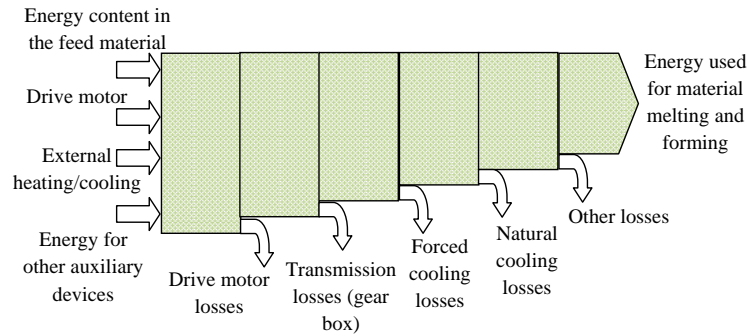


Figure 8: A typical energy flow diagram for an extruder [52]

extended for any auxiliary devices connected with the plant.

##### 4.1. Drive motor and gear box

The key component of any extrusion machine is the screw, which can be driven by a controllable direct current (DC) or an alternating current (AC) motor, or indeed by a hydraulic drive [10, 61]. The screw and the motor are connected through a gear box with fixed or adjustable transmission ratio, as shown in Figure 3. For the case of an extruder with a DC motor drive, see the schematic, presented in Figure 9. Additionally, the machine can have sensing and control devices related to its operation, for example, PID temperature controllers to control set barrel/die temperatures. Extruders with AC motor drives are essentially similar (with no rectifier), and may have additional components depending on the type of the motor.

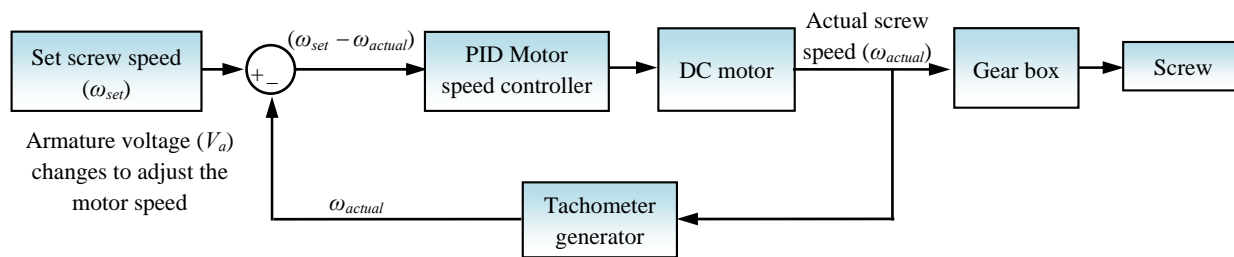


Figure 9: A schematic of an extruder drive mechanism

Figure 10 shows measured motor power and total power consumptions, for the case of a 63.5 mm diameter single screw extruder with a DC motor, driven at different screw speeds. The contribution of heaters to the total power demand is also indicated. As marked on Figure 10, all the heaters were turned off at around 330 s and this has led to smooth out the total power signal which were fluctuating due to the on-off action of the barrel/die heaters.

The drive motor is one of the major energy consuming components of an extruder [31, 34, 40], and, along with the gear box, is also responsible for significant energy losses, typically accounting for around 20% of the power supplied to an extruder [29]. In particular, DC motors are inefficient when operated at below the rated speed. Commercially available DC motors fall into three main categories: “permanent magnet”, “separately excited” and “self-excited”. The first two are more commonly used. A block diagram for a polymer processing extruder with a “separately excited direct current” (SEDC) motor is shown in Figure 11.

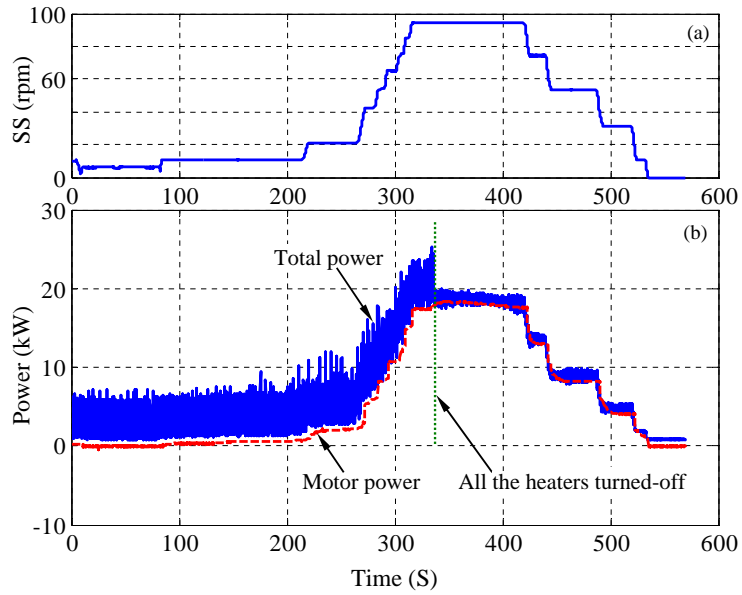


Figure 10: (a). Screw speed (SS), (b). Motor power and total power signals over the time [52]

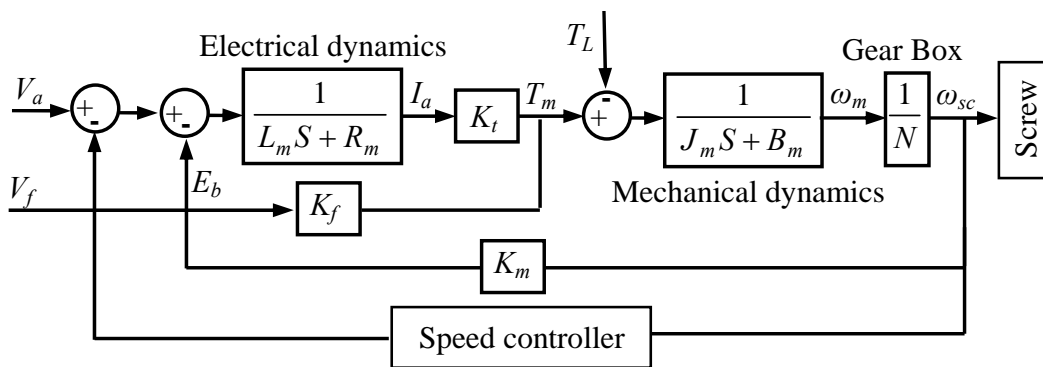


Figure 11: Block diagram of an extruder with a variable field DC motor [62]

In this figure,  $T_m$  is the motor torque,  $T_L$  is the load torque on the screw,  $R_a$  and  $L_a$  are the armature resistance and armature inductance respectively,  $K_f$  and  $K_t$  are the torque constants related to the field and motor, respectively,  $I_a$  is the armature current,  $V_f$  is the field voltage and  $B_m$  and  $J_m$  are the damping constant and the steady-state inertia of the loaded screw, respectively. For extruders with a permanent magnet motor, the same block diagram is valid without the branch related to the separately excited field (with  $V_f$  and  $K_f$  block).

Reasons for the popularity of DC motor drives in the polymer processing industry [63, 64, 52] include:

- Smooth operation over a wide speed range,
- Simplicity in speed control,
- Production of a constant/consistent torque from zero to base speed ,
- Relatively low power/energy consumption,
- Relatively smaller size compared to other drive types with the same capacity,
- Compact and simple power circuit, engaged with Silicon-controlled rectifiers,
- Easy installation,
- High reliability,
- Low initial capital cost, and
- Less noisy than AC motors.

Drawbacks of DC motor drives include the need for maintenance of brushes and commutator as well as the energy loss known as “brush loss” [59]. Green [63] observed that the best power factor that can be achieved by a DC motor operated at its top speed is of approximately 0.87, whereas for the best possible efficiency the power factor should be close to 1.

Currently, AC motor drives are increasing in popularity since the power factor can be maintained constant across the entire speed range. As a result, companies can avoid paying penalty charges for lagging power factor conditions. The most significant drawback of AC motors is that they require a constant current to produce a constant torque, hence demanding constant cooling regardless of the motor speed [63], and this results in an additional energy cost. Other issues include the fact that AC motors are generally larger, by a factor of 1.5-2.1, in volume, as well as being more complex than DC motors. These issues are becoming less critical nowadays, thanks to advances in electronics such as large scale chips and micro-processors. For both types of motors, while under operation at the rated speed, the maximum energy efficiency can be achieved, industrial extruders are typically operated at lower speeds in order to avoid undesirable fluctuations, particularly of the melt thermal quality.

Barlow [40] argues that because the displacement power factor of a DC motor drive is proportional to the speed, the power factor reduces as the motor slows down (see Figure 7). Further, it is pointed out that because the diode bridge of the input section of a pulse-width-modulated AC vector control drive rectifies the AC into DC, and that the energy is stored in capacitors, the current and voltage waveforms are mutually in phase, and hence the motor operates at a power factor, in the range of 0.90 to 0.98. More information on these motor drives can be found in the literature [65].

It seems that a significant amount of electrical energy may be lost simply as a result of the low power factor operation of motor drives [44]. Here, Eickelberg [66] suggests that use of capacitors may be one of the solutions to this problem, to smoothen the power supply. Several examples of the use of capacitors by commercial processes are provided, but it is stated that this is unlikely to be a practical solution for polymer extrusion processes because of the variability that occurs in the load. The installation of a more appropriate form of power factor correction would require investigation of the relevant issues [67].

Kent [41] observes that in extrusion plant energy usage assessments the energy requirements of motors in equipment such as extruders and injection moulding machines is often over-looked. Other authors [68, 69] report that considerable energy savings can be achieved by replacing DC motors with AC motors. Lounsbury and Karafilidis [70] present factors to be considered in the selection of a drive motor.

#### 4.2. Barrel and die heaters

Normally, three different types of heating method can be identified in extrusion. These are known as “resistance heating”, “induction heating” and “fluid heating” [61].

*Resistance heating:* This is also called *electrical heating*, and is the type most frequently used in extruders. Usually, electric heaters offer several advantages over fluid and steam heating, such as the possibility of covering a broader temperature range, cleanliness, easy maintenance, low cost, and better efficiency. As a result of these advantages, fluid and steam heaters have been replaced by electric heaters in modern applications. Currently extruders typically have between two and ten heating zones depending on the size of the extruder.

In the most common conventional resistance heaters, the heat from the resistance wire is transferred to ceramic segments that surround the outer surface of the barrel. This heats the barrel up until its inner surface is hot: the heat is then transferred to the plastic in the machine so it can be processed. With this heating method, much of the heat generated is wasted.

*Induction heating:* In this case, an AC current is passed through the primary coil surrounding the extruder barrel. This gives rise to an eddy current. Where the material being processed has significant relative permeability, heat may also be generated by magnetic hysteresis. A high energy density can be achieved quickly with induction heating. The frequency of the AC is selected depending on the size and material type and the heat penetration depth.

With the advantage of rapid heating and energy efficiency, induction heating has been used in many industrial applications [71]. The key advantage of induction heating over resistance heating is that the extruder barrel itself becomes the heating element. This eliminates the conduction problem that exists with conventional heaters. As there are no ceramic layers, clamping bands, or water jackets to heat up, the heat is direct and instantaneous. Furthermore, the induction heating generates a very even and precise heat profile, ensuring consistent heating of the polymer melt and leading to improved product quality. The application of induction heating to polymer processing has been developed by the Nordson Xaloy Company, which claims a reduction in heating related energy consumption of up to 50% compared with typical band electric heaters [72, 73].

*Fluid heating:* Fluid heating uses hot liquid or steam passing through pipes/tubes. This can ensure even temperature distribution but has significant disadvantages, such as demanding high levels of maintenance, the possibility of leaking or corrosion, and system complexity.

An alternative all three heating methods, is to supply the energy to the material being processed as drive power rather than as heater power [44, 74]. In this way shearing of the material leads to both heating and mixing. Of course, direct heating and cooling can be essential, to maintain the process thermal stability, but as guiding principles it would be preferable to minimize the direct heating and to avoid exceeding the melt temperature, in order to avoid unnecessary energy costs.

#### 4.3. Control electronics and monitoring devices

A number of control and monitoring devices are used in extrusion lines, such as speed and temperature controllers and indicators; pressure indicators and gauges; dimension scanners; feed monitoring devices; current indicators; relays; switches; alarms; *etc.*. All of these use some power for their operation; however, this is insignificant compared to the total energy demand. This is evident from the experimental data presented in Figure 10-(b), the amount consumed by control electronics and monitoring devices are shown by the difference between blue and red lines (i.e., the difference between total and motor powers) after turning off all the heaters, or by the blue line roughly between 540-570 s after turning off all heaters and the motor.

#### 4.4. Auxiliary equipment

Auxiliary equipment, such as pelletizers, gear pumps and screen changes, might also consume enough power to be considered in energy evaluations. Rice [75] suggests that improved energy efficiency for the entire operation of the plant can be achieved by combining processes.

#### 4.5. Process cooling

Process cooling is required where there is a need to remove excess heat in order to maintain the process thermal stability. This can be achieved by fan coolers attached along the barrel, or by cooling the screw core or the barrel wall internally, using a cooling fluid such as water or oil. Air cooling provides slower changes in temperature compared with liquid cooling.

Although cooling helps to ensure stable process operation, improper cooling, particularly cooling of the screw, can lead to the generation of undesirable process fluctuations. Strauch [31] argues that excessive screw core cooling can lead to a reduction in throughput rates, while at the same time affecting the melt temperature. This also impacts on the pumping stability as a direct consequence of altering the viscosity of melt. Periodic or random temperature variations of the extruder metal surfaces can arise as a result of cooling problems with the screw or barrel, and these may lead to melt flow problems such as melt viscosity fluctuations [76].

The research of Womer *et al.* [35] into the effects of cooling indicated that, where water cooling is used rather than air cooling, the extruder consumes more energy, irrespective of the material being processed. In consequence, it is recommended to use air cooling only, in conjunction with a properly designed screw, unless extensive cooling is required.

## 5. Trends in polymer processing energy efficiency improvements

### 5.1. Machine development and operational modifications

Among the current research and development “hot topics” for process energy efficiency, the three most likely to deliver significant benefits [63] are: (i) the design of direct drive extruders, (ii) improvements in heater and barrel design to reduce heat losses, and (iii) the development of “advanced vector control alternating current (AC) drives”. In addition to these, there are reports which claim that direct drive machines offer further advantages including energy efficient operation, narrow footprint, quiet operation, and low maintenance requirements. The use of insulation blankets has also become popular in energy saving of machines.

“Load management” offers the possibility to save power in the near term. The idea is to achieve the required energy cost reduction by maintaining the load factor, rather than focusing on power consumption *per se*. Since commercial electrical energy supply rates usually depend on the peak demand made by the customer, this can reduce the cost of the electricity used over the duration of the charging period. To operate an effective load management plan, it is necessary to have a plant monitoring system to study the real-time plant power usage. In Kent’s [41] discussion of energy saving in polymer processes, it is pointed out that the purchase of energy efficient capital equipment is profitable in long-run despite initial capital costs.

### 5.2. Waste heat energy recovery

Although a significant amount of process heat is removed purposely to maintain the thermal stability in polymer extrusion, there has been insufficient attention on the recovery of waste heat for useful work.

The challenge [42, 77] is to find a means of re-use for that energy. Future research is required to explore such opportunities. For example, the recovered heat might be used for pre-heating the material prior to feeding into the hopper, or it could simply be used for space heating.

For some materials, resins need to be pre-heated prior to processing to remove moisture. In this case, part of the supplied energy is lost through the evaporation of the moisture as water vapour, part is lost to heating of the surroundings, and the rest contributes to heating the resin. If the drying operation takes place remotely from the processing machine, then the heat absorbed by resin will be lost during transit to the processing machine. Therefore, the re-design of the drying system, as an in-line step of the processing machine, should help to reduce overall processing energy costs.

### 5.3. Material choice, material recycling and disposal

It is usually the case that the manufacture and fabrication of plastic products makes lower energy demands than equivalent traditional metallic or glassware products. The development of new resins that can be processed at lower temperatures, and hence for reduced energy, is part of the growing tide of interest to cut process energy expenses even further [78, 79, 80, 81]. On the other hand, polymeric materials based waste management has become a global concern. A wide range of recycling techniques are available depending on the types of polymers and the manufacturing techniques used [82]. In regard to process energy costs, the energy consumption for plastics is lower than for materials such as paper, glass, tin, and aluminium. What is more, Rosato *et al.* [34], the incineration of plastics as part of municipal waste yields much more energy than other material waste, such as food waste, paper and rubber, and waste volume can be reduced by 90-98%.

## 6. Advanced process monitoring and control: adoption of Industry 4.0

Advanced process monitoring and control can play a vital role in achieving good product quality as well as in energy optimization. Some approaches are discussed in this section together with experimental results.

### 6.1. Industry 4.0 and Internet of Things

Widespread uptake of on-line or in-line monitoring and control in manufacturing processes has been enabled by computerised communications, earning it the epithet: “The fourth industrial revolution”, or “Industry 4.0” for short [83, 84]. The key requirements for an Industry 4.0 manufacturing process are: *sensors* - the means to observe the process as it currently stands; *actuators* - the means to modify the process; *electronic communications* - the means to pass sensor or control information; and a *decision-maker* - to determine the course of action to be taken based on the information received.

The precise nature of the decision-maker is a point of some contention in the literature. For some, the decision-maker is a computer-based Artificial Intelligence (AI), which would work completely autonomously, and steadily improving its decision-making capability based on learned patterns of experience. There is still significant value in the Industry 4.0 infrastructure even without an AI capability making the control decisions. For some applications, a rule- or model-based computer program would be effective, and many companies now boast of having an Industry 4.0 implementation of this form. In some applications, having the sensor information fed to a control centre, means that human decision-making can be facilitated and supported. Such control centres are valuable for the control of processes that are in remote or difficult to access locations, such as the health-monitoring of in-flight aircraft.

Industry 4.0 technology is also becoming an increasingly common tool in the home or in social care settings. Here, the more common terminology is Internet of Things (IoT) [85], referring to communications connectivity between “Smart” devices. These smart devices are pieces of equipment which can send or receive information, in other words, they are equipped with sensors or actuators. Thus, in the home, one might simply ask “Alexa” [86] to turn on the lights, but in time it could easily be imagined that Alexa could modify the home heating to match the schedule inferred from family member diaries. In a patient care setting [87], the IoT system could be collecting valuable health-related information and relaying that to a control centre. Care staff or an AI system might detect anomalies and prompt a check of the patient. It is easy to see that these concepts are similar, whether applied to the home or to industry, and the technology is pervasive and rapidly developing.

In the context of the polymer processing industry, it is clear that Industry 4.0 will not only enhance plant operation and its maintenance schedule, but also to energy efficiency [88, 89]. The lessons learned over the past half century, and reviewed in earlier sections of this paper, are ready to be applied. Wherever there are frequent variations in the feedstock material, processing rate demand, or other factors, with an Industry 4.0 infrastructure in place, it becomes possible to monitor performance over time, and to make controlled changes. Systems health monitoring can be used to reduce life limiting loads on mechanical parts [90] as part of the maintenance strategy.

Greater investment in IoT enabled devices will become an increasing imperative, for any polymer processing plant that wishes to reduce its energy footprint, reduce processing costs, and sustain the mechanical plant more effectively. As soon as the Industry 4.0 infrastructure is operational, development of the decision-making capability can be begun; whether that is to be based on AI principles, control systems mathematics or other rule or model based paradigms.

### 6.2. Ultrasound

The application of ultrasonic waves to reduce viscosity and thus energy consumption in polymer processing has been discussed in several recent reports [91, 92, 93, 94, 95, 96]. Maintaining a consistent melt viscosity enables improved process ability, leads to fewer product defects, reduces energy consumption and reduces materials wastage [97, 98, 99].

Chen *et al* [92] and Zhang and Li [93] report on the use of ultrasound vibration to influence polypropylene (PP) melt, leading to non-Newtonian flow characteristics with reduced viscosity. Other authors have examined the relationships between temperature, pressure, work-stuff throughput, the energy consumption and the ultrasonic intensity [95, 96]. The integration of ultrasound into a closed loop extruder control system has been developed and introduced in detailed by Nguyen *et al.* [100], who showed that controlling just the temperature, or just the ultrasonic output, gave a better time response and reduced energy consumption, than when trying to control both the temperature and ultrasound together.

### 6.3. Closed loop melt temperature control

Melt temperature can serve as a proxy parameter for melt viscosity and can thus be used to determine the melt quality. Recent works by Abeykoon et al [101, 102, 103] incorporate a fuzzy logic approach for the real-time closed loop control of melt temperature, and demonstrated excellent performance in achieving desired set temperatures.

At present, in the majority of polymer processes, melt pressure and melt temperature are taken as the key parameters for process functionally and control. Screw speed, and barrel and die set temperatures are taken as the main process control parameters, but there is no actual feedback taken from the process melt for making process control decisions, so no corrective actions can be taken to avoid product defects. Furthermore, this affects the production rate leading to wasted energy, labour and raw materials. Hence, combined process monitoring and control approaches, which can observe the melt quality and take control actions would represent a major development of polymer processes.

## 7. Applying computational process simulation and dynamical systems to Control

Developments in computational process simulation methods has the potential to revolutionise the design of manufacturing tooling and processes. Capabilities such as finite element method and computational fluid dynamics have developed very significantly, with most commercial packages (*e.g.* [104, 105, 106, 107, 108, 109]) now offering multi-physics simulation: simulation of static and transient solid mechanics, fluid dynamics, thermodynamics, electromagnetism, and in many cases, much more. Given the improvements in computer hardware, storage capacity and the development of parallel processing architectures, computational analyses with high levels of geometric or material modelling complexity can now be readily envisaged. This is a huge opportunity to grasp, and one for which most industrial plants are not fully prepared.

### 7.1. Computational mechanics methods

The key computational capabilities pertinent to polymer processing include a variety of modelling techniques for viscoelastic materials irrespectively of whether they should be treated as solids or liquids. Where previously, modelling based on a Lagrangian Finite Element formulation, [110], would have been unable to capture the necessary deformation, Eulerian formulations, [111], and Smooth Particle Hydrodynamics (SPH), [112] and [113], now have that potential. Perhaps it is not enough to model using just one technique or another, but to capture one pertinent aspect of the physics in one region of the process, and another aspect in another region using such modelling techniques as co-simulation and subdomain modelling.

Not only it is the type of analysis that is of importance to capture the process simulation requirement, but also the way in which the material properties are represented. The distinction between solid and liquid is no longer so clear-cut. There are very many material models, developed and applicable to different analysis types, that will capture the essential material property physics of any realizable material. Polymers offer particular challenges, but characteristics such as static stress–strain, strain rate dependence, creep, and temperature dependence are readily modelled. Chemical changes, including exo– or endothermic reactions, and cure shrinkage still present a particular challenge to thermoset polymers, but for thermoplastic processing the challenge of representing materials properties at glass transition, and extent of crystallization, are, if not straight forward, at least more tractable [114, 115, 116]. For the characterisation of polymers undergoing extrusion processing Abeykoon *et al.*, [117], have made significant investigations.

With the increase of computing power and the capability for higher model complexity there has been increased interest in the application of computational mechanics methods using sophisticated material models to the simulation of the extrusion process. In the past decade authors such as Zairi *et al.*, [118], have been able to model plastic flow, taking account of viscoelastic properties, within a die of finite constrained section, and have been able to make some prediction of the microstructure of the end product. While the main focus has been of the effect of the die geometry on the extruded product, another important consideration is the design of the die tooling, and the loads that must withstand during processing [119].

### 7.2. Statistical process control

The biggest challenge for the modelling of the polymer extrusion process is to understand the conditions that give rise to large variations in product output quality. Where small changes in the processing parameters lead to only small changes in output, even where the change is clearly non-linear, one might take a step-wise approach to linearize the parameters and have a measurable and definable set of limits for process control. This is the ideal, textbook, approach to manufacturing process excellence, LEAN manufacturing or Six Sigma, [120], [121]. In contrast, where small changes in processing input lead to significant changes in the output quality, a manufacturing engineer would say that the process is not in statistical control. The usual manufacturing engineer's approach for regaining statistical control is to monitor parameters to within ever tighter bounds. Clearly this would lead to increased manufacturing costs, and could make the process financially unviable. The approach may ultimately even be completely unsuccessful.

The factors that would generally be considered for statistical process control would include the initial conditions: how the process is started up, and the material condition on start-up, factors directly related to the geometry of the extruder and the die, environmental conditions and changes, and changes in the raw material batch.

In order to control the process, certain actions might be taken. The applied drive torque, a function of the power supplied to the extruder, could be controlled in direct response to measurements relating to the process quality. Inevitably there is a delay between making the measurement, and making a change, primarily because the measurement is taken at some point down-stream in the process. There is a considerable level of research activity in this area, with various computational schemes for assessing these measurements and informing the choice of control action to be taken. Clearly this data processing time must be minimised to minimise the feedback delay. Wagner *et al*, [122], and McKay *et al*, [123], recognised that the fundamental requirement was to use the measurements to infer the local material viscosity within the process, using artificial intelligence methods such as neural nets. Chen *et al*, [124], employed a power law model. Methods based on fuzzy logic were developed by McAfee, [125], McAfee and Thompson, [126], and later by Liu *et al*, [127] and Abeykoon [103]. More recent work includes the soft sensor technique of Deng *et al*, [128] and Abeykoon [102], and the multi-objective optimisation approaches presented by Carrano *et al*, [129].

### 7.3. Non-linear system dynamics

Where the dynamics of the manufacturing process make statistical process control challenging, a radically different approach is needed. Non-linear systems dynamics [130] has been a topic of considerable research and development, mainly by researchers with a strong background in applied mathematics, and with applications including but by no means limited to manufacturing engineering applications. By developing a non-linear system dynamics representation of a manufacturing process, it is possible to explore how process parameters could influence the process dynamics, and from this pin-point the controlling factors or initial conditions.

In the field of polymer extrusion modelling, McKinley, *et al*, [131], used laser doppler velocimetry to visualise the flow towards an abrupt contraction, and found that the flow near the tip of the contraction could show time periodic and aperiodic behaviour. Graham [132] modelled the fluid behaviour regarding wall slip and was able to replicate the large amplitude periodic and aperiodic oscillations observed experimentally. Smith *et al*, [133, 134] demonstrate a steady-state solution through modelling the polymer as a purely viscous non-Newtonian material, and optimising die geometry.

The critical feature in the extrusion process is the variation in the material property of the polymer as it passes through the process. As the material is being worked mechanically, it becomes heated. As it gets hotter, the elastic stiffness and viscosity are reduced. The resulting thermal expansion gives rise to a localised increase of pressure. As a result of both the reduced viscosity and increased pressure, the polymer would pass more readily through the process: and would require less working. Reduced working would result in reduced heating, with the result of increased stiffness, increased viscosity and reduced pressure. It is clear that a cyclic response can be expected. Now consider how the viscosity of the polymer changes with strain rate and the difference of time dependency in glass transition and crystallization, it becomes clear that there is a level of complexity between production rate, localized temperature and viscous response of the material: in the simplistic mass, spring and damper system it is the variability of the damper as well as the forcing that is key to understanding the dynamical system.

A non-linear dynamical systems model of the polymer extrusion process might be modelled as follows. First, the polymer has the properties of both an elastic solid, and a viscous fluid: it can be idealized as a mass, spring and damper system, as shown in Figure 12. The non-linear dynamics of such systems have been studied for many



idealized situations of varying applied force,  $F(t)$ , and their modelling involves the selection of initial conditions, and then computing the transient behaviour until a long term behaviour becomes apparent.

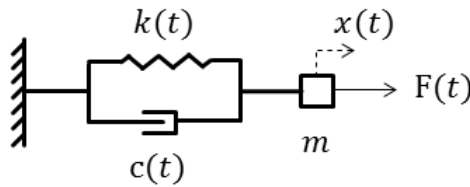


Figure 12: Idealised mass, spring and damper system

Figure 13 shows the time varying position,  $x(t)$  and velocity  $\dot{x}(t)$  of the mass, driven by the time varying force  $F(t)$ . The initial conditions for the system might be any point on the  $x - \dot{x}$  plane, and one might think of particular examples of such initial conditions being the tail end of the solid or dashed curved arrows. As time progresses, the system would move on from that initial condition, moving towards the head of the arrow. The dashed arrows indicate the evolution away from an unstable limit cycle – the dashed closed loop. Any initial condition that begins on the unstable limit cycle will eventually decay inwards or outwards, and migrate towards a stable limit of some sort. The solid arrows indicate evolution towards either a stable limit cycle – the solid closed loop, or towards a steady-state solution – the solid dot. The stable limit cycle represents a periodic motion. The area enclosed by the unstable limit cycle gives an indication of the relative level of damping in the system: it is important to note that increasing the damping would increase the area within the unstable limit cycle, so that a larger proportion of initial conditions would ultimately lead to a steady-state solution; however in the case of initial conditions sufficiently close to the stable limit cycle, periodic motion would still ensue. The notion that damping removes energy from a dynamic system is true, but the energy required for periodic motion is renewed at every cycle by the time varying force,  $F(t)$ .

In the context of polymer processing, periodic motion represents both unnecessary energy utilisation, and reduced process control. This explanation is more easily understood when described in the context of everyday experience.

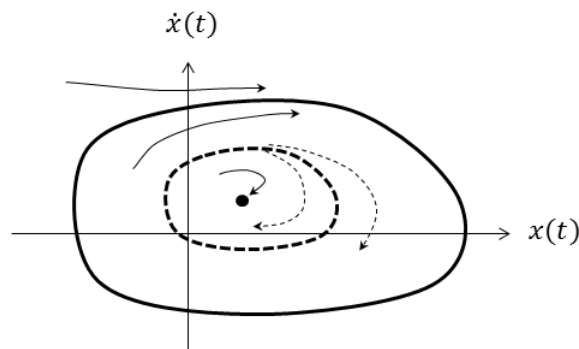


Figure 13: Time evolving position and velocity diagram

Consider a friction driven vibration such as the vibration of a bowed violin string, brake squeal, or a singing wine glass. In each case, the string, the brake disc and the glass have a stiffness, a density and some form of damping is present. Under particular forcing conditions such as applied by a well-rosined violin bow, a certain braking pressure or being stroked with a wet finger, these structures vibrate at a well-defined pitch. The fact that the pitch is well-defined means that the vibration is periodic. Under different forcing conditions: a bow without rosin, a different braking pressure or a dry finger, the contact slips with constant velocity. This is a steady-state solution: a force is being applied and so there is a displacement, but it is a constant force and a constant displacement. For a detailed account of such an analysis, and the computational algorithms used [135].

This description is necessarily rather simplistic, but it provides a basis for reviewing the pertinent features required for modelling the extrusion system. In place of displacement and velocity one should be considering the extruded volume of material as a function of time. In place of force, consider the power output of the machine and what resistance it meets from the material being processed. In an ideal process, one would wish for constant extruded volume and constant levels of resistance, so that with an appropriate choice of initial conditions and damping, a steady-state outcome can be achieved. The elastic response and the viscosity of the polymer depend on temperature and strain rate, and as such are the time dependent system stiffness,  $k(t)$ , and time dependent damping,  $c(t)$ , respectively. Recent work in this vein, but applied to blown film extrusion, is presented in a detailed review paper by Pirkle *et al.*, [136].

## 8. Conclusions

There have been some very significant developments in computational modelling capability, which means that the simulation of manufacturing processes such as polymer extrusion is now feasible. Multi-physics approaches, combining material flow and thermal behaviour of the material can, in theory, be modelled. Advances in computer hardware mean that models with very high levels of geometric complexity, and therefore high numbers of computational degrees of freedom are within reach. Co-simulation modelling, including the modelling of not only the material undergoing extrusion, but also of the working state of the dies and the extrusion machine itself, is also within reach, meaning that optimal design of tooling for improved life and reduced machine maintenance demands are additional areas for further development.

Complex models of the process and material property variation can also be used to create the simplified models required for a non-linear systems dynamics modelling approach. On that basis, measurable parameters that would have an effect on the stability of the real production process can be examined, and warning limits found. Even where such an approach might not be able provide an accurate prediction of limits, it could provide insight that would lead to practical solutions or avoidance of particular operating regimes.

## 9. List of Abbreviations

<b>Term</b>	<b>Definition</b>
AC	Alternating current
DC	Direct current
IoT	Internet of Things
SEC	Specific energy consumption
SEDC	Separately excited direct current

## 10. List of Symbols

<b>Term</b>	<b>Definition</b>
$B_m$	Damping constant
$C_p$	Specific heat capacity
$E_{in}$	Energy consumed
$E_{losses}$	Energy loss
$F(t)$	A time varying force
$I$	Line current
$I_a$	Armature current
$I_f$	Field current
$J_m$	Steady-state inertia of the loaded screw
$k_m$	Thermal conductivity
$K_f$	Torque constant related to the field
$K_t$	Torque constant related to the motor
$L_a$	Armature inductance
$N$	Gear ratio
$P$	Active power
$Q$	Reactive power
$R$	Electrical resistance
$S$	Apparent power
$R_a$	Armature resistance
$T_b$	Extruder barrel set temperature
$T_L$	Load torque on the screw
$T_{melt}$	Melt temperature
$T_m$	Motor torque
$x(t)$	A time varying position
$\dot{x}(t)$	A time varying position velocity
$V$	Line voltage
$V_a$	Armature voltage
$V_{bx}$	The component of the barrel velocity in the transverse direction
$V_f$	Field voltage
$V_j$	The resultant relative velocity

Term	Definition
$\cos \phi$	The displacement power factor
$\rho_m$	Melt density
$\eta$	Melt viscosity
$\omega_{actual}$	Actual screw speed
$\omega_{set}$	Set screw speed
$\lambda$	Temperature of the solid bed
$\Omega$	Rate of melting

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